Six texture Zeros for the SM quark mass matrices

William A. Ponce
Instituto de Física.
Universidad de Antioquia.
Medellín Colombia.

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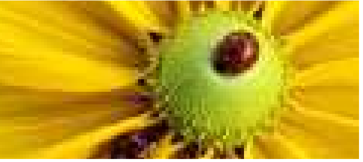


Flavor problem

$$-\mathcal{L}_M = \bar{U}_{0L} M_u U_{0R} + \bar{D}_{0L} M_d D_{0R} + h.c,$$

$\bar{U}_{0L} = (\bar{u}_0, \bar{c}_0, \bar{t}_0)_L;\ \ \bar{D}_{0L} = (\bar{d}_0, \bar{s}_0, \bar{b}_0)_L$: Weak basis.

Sector poorly understood

- Total number of families?
- Hierarchical quark masses?
- Small neutrino masses?
- Quark and lepton mixing angles?
- Origin of CP violation?

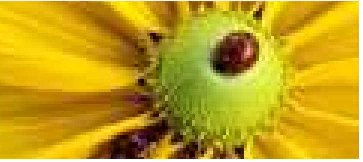
Radiative mechanisms? Horizontal symmetries?
Froggat-Nielsen? Texture Zeros?

Mass Matrices

$$M_u \quad M_d$$

two 3×3 complex mass matrices
36 free parameters.

POLAR THEOREM OF MATRIX ALGEBRA
$M = H.U$ with $H^\dagger = H$ and $U^\dagger = U^{-1}$.
U absorbed in U_{0R} and D_{0R} in SM.
$\Longrightarrow$ down to 18 free parameters. 12 real and 6 phases.
only one phase with physical meaning.
$\Longrightarrow$ down to 12 real parameters and 1 phase.



Charged current

$$J^{-}_{\mu L} = \bar{U}_{0L}\gamma_\mu D_{0L} = \bar{U}_L \gamma_\mu V_{CKM} D_L,$$

$$V_{CKM} = U_u U_d^\dagger$$

$\bar{U}_L = (\bar{u}, \bar{c}, \bar{t})_L$ and $D_L^T = (d, s, b)_L$

stand for the quark field, mass eigenstates

- U_u diagonalize the Hermitian $M_u M_u^\dagger$
- U_d diagonalize the Hermitian $M_d M_d^\dagger$

Weak basis

A unitary transformation acting simultaneously in M_u and M_d does not change V_{CKM}.

$$M_u \longrightarrow M_u^R = U M_u U^\dagger$$
$$M_d \longrightarrow M_d^R = U M_d U^\dagger$$

$$V_{CKM}^R = U_u^R U_d^{R\dagger} = U_u U^\dagger U U_d^\dagger = U_u U_d^\dagger = V_{CKM},$$

3 texture zeros without physical meaning.

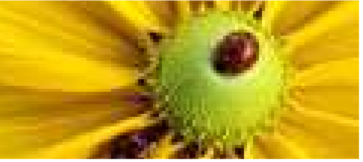
$$(M_u^R)_{11} = (M_d^R)_{11} = (M_u^R)_{13} = (M_u^R)_{31} = 0.$$

$\Longrightarrow$ down to 9 real parameters and 1 phase.

To explain 6 quark masses, 3 mixing angles and CP violation.

One more texture zero $\rightarrow$ New physics.

G.C.Branco, D.E.Costa and R.G.Felipe, Phys. Lett B477, 147 (2000).



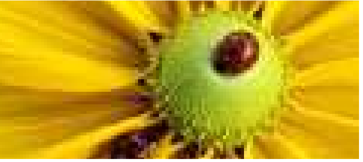
Parallel 6 zeros.

$$M_q^{(6)} = \begin{pmatrix} 0 & a_q e^{i\alpha_q} & 0 \\ a_q e_q^{-i\alpha_q} & 0 & b_q e^{i\beta_q} \\ 0 & b_q e^{-i\beta_q} & c_q \end{pmatrix}, \qquad (1)$$

where q stands for u and d.

$$\begin{aligned} \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_u} & 0 \\ 0 & 0 & e^{-i(\alpha_u+\beta_u)} \end{pmatrix} \begin{pmatrix} u_0' \\ c_0' \\ t_0' \end{pmatrix} \\ &= U_u^\dagger \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}, \end{aligned}$$

H.Fritzsch, Phys. Lett. B**73**, 317 (1978)



Orthogonal rotation

$$M_q^{(6)\prime} = \begin{pmatrix} 0 & a_q & 0 \\ a_q & 0 & b_q \\ 0 & b_q & c_q \end{pmatrix}, \quad (2)$$

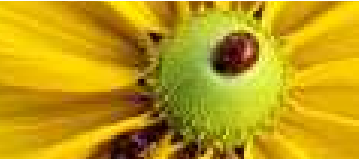
Using the invariants $\mathrm{tr} M_q^{(6)\prime}$, $\mathrm{tr}[(M_q^{(6)\prime})^2]$, $\det M_q^{(6)\prime}$.

$$c_q = m_1 - m_2 + m_3$$

$$a_q^2 = \frac{m_1 m_2 m_3}{m_1 - m_2 + m_3}$$

$$b_q^2 = \frac{(m_3 - m_2)(m_3 + m_1)(m_2 - m_1)}{m_1 - m_2 + m_3}.$$

$$O_q^{(f6)} = \left(\begin{array}{ccc} \pm\sqrt{\frac{m_2 m_3 (m_3 - m_2)}{(m_3 - m_1)(m_2 + m_1)(m_1 - m_2 + m_3)}} & \pm\sqrt{\frac{m_1 m_3 (m_1 + m_3)}{(m_2 + m_1)(m_3 + m_2)(m_1 - m_2 + m_3)}} & \pm \\ \pm\sqrt{\frac{m_1 (m_3 - m_2)}{(m_3 - m_1)(m_1 + m_2)}} & \mp\sqrt{\frac{m_2 (m_1 + m_3)}{(m_2 + m_3)(m_1 + m_2)}} & \\ \mp\sqrt{\frac{m_1 (m_2 - m_1)(m_1 + m_3)}{(m_3 - m_1)(m_1 + m_2)(m_1 - m_2 + m_3)}} & \pm\sqrt{\frac{m_2 (m_3 - m_2)(m_2 - m_1)}{(m_2 + m_1)(m_3 + m_2)(m_1 - m_2 + m_3)}} & \pm \end{array} \right. \quad (3)$$



Mixing Matrix I

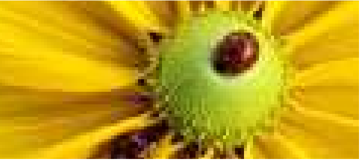
$$\begin{aligned}(V_{CKM}^{(f6)})_{lm} &= (O_u^{(f6)})_{l1}(O_d^{(f6)})_{m1} \\ &+e^{i\phi_1}(O_u^{(f6)})_{l2}(O_d^{(f6)})_{m2} \\ &+e^{i\phi_2}(O_u^{(f6)})_{l3}(O_d^{(f6)})_{m3},\end{aligned} \tag{4}$$

where $\phi_1 = (\alpha_u - \alpha_d)$ and $\phi_2 = (\alpha_u + \beta_u - \alpha_d - \beta_d)$.

$$\begin{aligned}V_{us}^{(f6)} &= (1 - \frac{m_{uc}}{2}) \left[\sqrt{\frac{m_{ds}(1-m_{sb})}{(1-m_{db})(1+m_{ds})}} \right. \\ &- \left. e^{i\phi_1}\sqrt{\frac{m_{uc}(1+m_{db})}{(1-m_{sb})(1+m_{ds})}} \right] \leq 0.178,\end{aligned} \tag{5}$$

Compared with $V_{us}^{exp} \approx \cos_{\theta_c} = 0.223$

Ruled Out



Non-Parallel 6 zeros

$$M_u^{(np)} = \begin{pmatrix} 0 & a_u e^{i\alpha_u} & 0 \\ a_u e^{-i\alpha_u} & 0 & b_u e^{i\beta_u} \\ 0 & b_u e^{-i\beta_u} & c_u \end{pmatrix}. \tag{6}$$

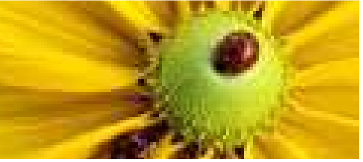
$$M_d^{(np)} = \begin{pmatrix} 0 & a_d e^{i\alpha_d} & 0 \\ a_d e^{-i\alpha_d} & b_d & 0 \\ 0 & 0 & c_d \end{pmatrix}, \tag{7}$$

θ_{13} and θ_{23} came from the up quark sector only.

The up sector same as before.
For the down setor, the invariants imply

$$c_d = m_b \qquad b_d = m_d - m_s < 0$$

$$a_d = \sqrt{m_d m_s}$$



Mixing Matrix II

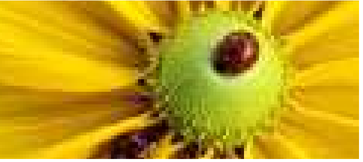
$$\begin{aligned}(V_{CKM}^{(f6)})_{lm} &= (O_u^{(f6)})_{l1}(O_d^{(np)})_{m1} \\ &+e^{i\phi_1}(O_u^{(f6)})_{l2}(O_d^{(np)})_{m2} \\ &+e^{i\phi_2}(O_u^{(f6)})_{l3}(O_d^{(np)})_{m3},\end{aligned} \tag{8}$$

where $\phi_1 = (\alpha_u - \alpha_d)$ and $\phi_2 = (\alpha_u + \beta_u - \alpha_d - \beta_d)$.

$$O_d^{(np)} = \begin{pmatrix} \sqrt{\frac{m_s}{m_d+m_s}} & -\sqrt{\frac{m_d}{m_d+m_s}} & 0 \\ \sqrt{\frac{m_d}{m_d+m_s}} & \sqrt{\frac{m_s}{m_d+m_s}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

$$V_{us}^{(np)} = \frac{(1 - m_{uc}/2)}{\sqrt{m_d + m_s}} \left[\sqrt{m_d} + e^{i\phi_1}\sqrt{m_{uc} m_s}\right], \tag{10}$$

which for $\phi_1 = 1.49$ produces a value $0.222 \leq V_{us}^{(np)} \leq 0.223$



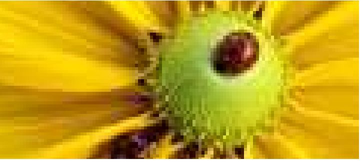
Experimental values

Meassured at the M_Z mass scale

Up sector	Down sector
$m_t = 171.7 \pm 3.0$ GeV	$m_b = 2.89 \pm 0.009$ GeV
$m_c = 0.619 \pm 0.084$ GeV	$m_s = 55^{+16}_{-15}$ MeV
$m_u = 1.27^{+0.5}_{-0.42}$ MeV	$m_d = 2.90^{+1.24}_{-1.19}$ MeV

$$V^{(exp)} = \begin{pmatrix} 0.970 \leq |V_{ud}| \leq 0.976 & 0.223 \leq |V_{us}| \leq 0.228 & 0.003 \leq |V_{ub}| \leq 0.004 \\ 0.217 \leq |V_{cd}| \leq 0.237 & 0.960 \leq |V_{cs}| \leq 0.990 & 0.039 \leq |V_{cb}| \leq 0.040 \\ 0.008 \leq |V_{td}| \leq 0.009 & 0.038 \leq |V_{ts}| \leq 0.042 & 0.78 \leq |V_{tb}| < 1.00 \end{pmatrix}$$

$$\begin{aligned} \alpha &\equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] = 91.0 \pm 7.2 \\ \beta &\equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] = 21.8 \pm 2.8 \\ \gamma &\equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right] = 67.2 \pm 9.1 \end{aligned}$$



Numerical results

$$V_{CKM}^{(np)} = \begin{pmatrix} 0.970 & 0.226 & 0.003 \\ 0.225 & 0.965 & 0.039 \\ 0.008 & 0.038 & 0.997 \end{pmatrix},$$

$$(\alpha,\ \beta,\ \gamma)_{th}^{(np)} = (93.90, \quad 15.48, \quad 70.62)$$

All agree but β