Chapter 4

Two body decays

In this chapter we assume the Feynman rules for Fermions to carry out the calculation of the decay of the standard model Higgs into a pair of fermions. In chaper 5 we will derive the corresponding Feynman rules from the S–matrix expansion.

4.1 Particle decays

Particle decay [5] is the spontaneous process of one elementary particle transforming into other elementary particles. During this process, an elementary particle becomes a different particle with less mass and an intermediate particle such as W boson in muon decay.

For a particle of a mass M, the differential decay width according Eq. (3.36), is

$$d\Gamma_n = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2 \, d\Phi^{(n)}(P; p_1, p_2, \ldots, p_n) \tag{4.1}$$

The phase space can be determined from Eq. (3.35)

$$d\Phi^{(n)}(P; p_1, p_2, \ldots, p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \left(\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \right) . \tag{4.2}$$

We will keep the $d\Gamma$ notation until all the integrals get evaluated.

4.2 Width decay

For the $H \to f\overline{f}$ decay, Eq. (4.1) reads

$$
\begin{aligned}
d\Gamma &= \frac{1}{2m_{\mathcal{A}}}\frac{1}{4\pi^2}\frac{d^3\mathbf{p}_1}{2E_1}\frac{d^3\mathbf{p}_2}{2E_2}\delta^{(4)}(p_{\mathcal{A}}-p_1-p_2)|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2 \\
&= \frac{1}{2m_{\mathcal{A}}}\frac{1}{4\pi^2}\frac{d^3\mathbf{p}_1}{2E_1}\frac{1}{2E_2}\delta(E_{\mathcal{A}}-E_1-E_2)|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2 \\
&= \frac{1}{2m_{\mathcal{A}}}\frac{1}{4\pi^2}\frac{\mathbf{p}_1^2 d\mathbf{p}_1 d\Omega_{cm}}{4E_1E_2}\delta(E_{\mathcal{A}}-E_1-E_2)|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2, \qquad (4.3)
\end{aligned}
$$

From the energy and momentum conservation, implicit in $\delta^{(4)}$, we have $\mathbf{p}_1 = \mathbf{p}_2$. Therefore $E_{\mathcal{A}} = E_1 + E_2 = (m_1^2 + \mathbf{p}_1^2)^{1/2} + (m_2^2 + \mathbf{p}_1^2)^{1/2}$. In this way

$$
\frac{dE_{\mathcal{A}}}{d\mathbf{p}_1} = \mathbf{p}_1\left(\frac{1}{E_1}+\frac{1}{E_2}\right). \qquad (4.4)
$$

Replacing Eq.(4.4) back in Eq. (4.3), we have

$$
\begin{aligned}
d\Gamma &= \frac{1}{2m_{\mathcal{A}}}\frac{1}{4\pi^2}\frac{\mathbf{p}_1 dE_{\mathcal{A}} d\Omega}{4(E_1+E_2)}\delta(E_{\mathcal{A}}-E_1-E_2)|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2 \\
&= \frac{1}{2m_{\mathcal{A}}}\frac{1}{4\pi^2}\frac{\mathbf{p}_1 d\Omega_{\rm CM}}{4E_{\mathcal{A}}}|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2
\end{aligned}
$$

Finally,

$$
\frac{d\Gamma}{d\Omega_{\rm CM}} = \frac{\mathbf{p}_1}{32\pi^2 m_{\mathcal{A}}^2}|\mathcal{M}(\mathcal{A}\to 1{+}2)|^2 \qquad (4.5)
$$

4.3 Feynman Rules and trace theorems

The interaction between the Higgs boson with fermions[1] is given by the Yukawa interaction term [1]

$$
\begin{aligned}
\mathcal{L}_{Yukawa} &= -G_f\frac{(v+\eta)}{\sqrt{2}}(\bar{e}_R e_L + \bar{e}_L e_R) \\
&= -\frac{G_f v}{\sqrt{2}}\bar{e}e - \frac{G_f\eta}{\sqrt{2}}\bar{e}e
\end{aligned}
$$

Such as the electro has acquired a mass $m_e = G_f\nu/\sqrt{2}$. On the other hand the coupling to be assigned to the process vertex is $G_f\sqrt{2}$ or m_e/v.

The decay process $H \to f\overline{f}$, is displayed in Fig. 4.1

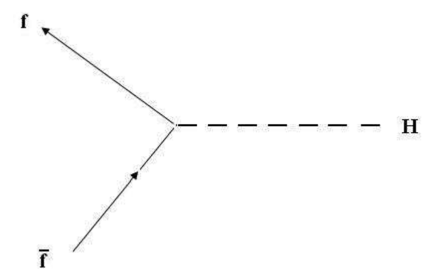

Figure 4.1: Diagrama de proceso $H \to f\overline{f}$

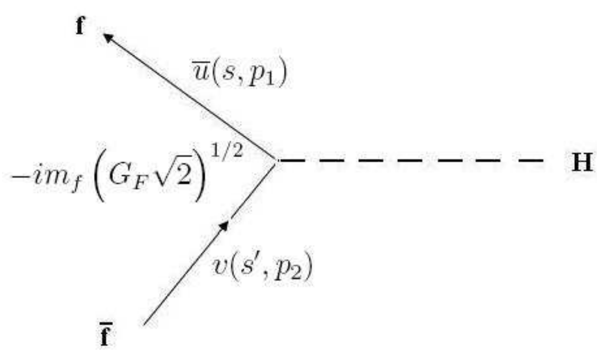

Figure 4.2: Reglas de Feynman del proceso $H \to f\overline{f}$

The Feynman rules, to be explained in Chapter 5 are indicated in Fig. 4.2.

In this way the scattering amplitude is

$$i\mathcal{M} = -im_f \left(G_F\sqrt{2}\right)^{1/2} \overline{u}(s, p_1) v(s', p_2). \tag{4.6}$$

where p_1, s, p_2 y s' are the momentum and spines of fermion and anti–fermion respectively.

[1] In this case we consider only electrons, by the formula is easy generalizable to other fermions

Now, having into account that $\gamma^{0\dagger} = \gamma^0$

$$
\begin{aligned}
&(\overline{u}(s,p_1)v(s',p_2))^\dagger \\
&= v^\dagger(s',p_2)(\overline{u}(s,p_1))^\dagger \\
&= v^\dagger(s',p_2)(u^\dagger(s,p_1)\gamma^0)^\dagger \\
&= v^\dagger(s',p_2)(\gamma^{0\dagger}u(s,p_1)) \\
&= v^\dagger(s',p_2)(\gamma^0 u(s,p_1)) \\
&= (\overline{v}(s',p_2)u(s,p_1)).
\end{aligned}
$$

Squaring $\mathcal{M}$, and summing over possible polarization states of final particles, we have

$$
\sum_{s,s'} |\mathcal{M}|^2 = G_F m_f^2 \sqrt{2} \sum_{s,s'} (\overline{u}(s,p_1)v(s',p_2))(\overline{v}(s',p_2)u(s,p_1)). \tag{4.7}
$$

s The several sums in Ec. (4.7) can be calculated by expressing the products $\overline{u}v$ y $\overline{v}u$ en in terms of their components, as follow

$$
\begin{aligned}
&\sum_{s,s'} (\overline{u}(s,p_1)v(s',p_2))(\overline{v}(s',p_2)u(s,p_1)) \\
&= \sum_{s,s'} (\overline{u}_\alpha(s,p_1)v_\alpha(s',p_2))(\overline{v}_\beta(s',p_2)u_\beta(s,p_1)) \\
&= \sum_{s,s'} (u_\beta(s,p_1)\overline{u}_\alpha(s,p_1))(v_\alpha(s',p_2)\overline{v}_\beta(s',p_2)) \\
&= \sum_{s} u_\beta(s,p_1)\overline{u}_\alpha(s,p_1) \sum_{s'} v_\alpha(s',p_2)\overline{v}_\beta(s',p_2) \\
&= (\not{p}_1 + m_f)_{\beta\alpha}(\not{p}_2 - m_f)_{\alpha\beta} \\
&= \mathrm{Tr}[(\not{p}_1 + m_f)(\not{p}_2 - m_f)].
\end{aligned} \tag{4.8}
$$

Taking into account that $\mathrm{Tr}[\gamma_\nu] = 0$, and from the commutation relations for γ_μ matrices

$$
\begin{aligned}
\mathrm{Tr}[\gamma_\mu\gamma_\nu] &= tr[-\gamma_\nu\gamma_\mu + 2g^{\mu\nu}] \\
&= \mathrm{Tr}[-\gamma_\nu\gamma_\mu] + 2g^{\mu\nu}\,\mathrm{Tr}[\mathbf{1}] \\
&= \mathrm{Tr}[-\gamma_\mu\gamma_\nu] + 2g^{\mu\nu}4 \qquad (\mathrm{Tr}[AB] = \mathrm{Tr}[BA]) \\
\mathrm{Tr}[\gamma_\mu\gamma_\nu] &= 4g^{\mu\nu}.
\end{aligned}
$$

In this way

$$\begin{aligned}
&\mathrm{Tr}[(\not{p}_1 + m_f)(\not{p}_2 - m_f)] \\
&= \mathrm{Tr}[(\gamma_\mu p_1^\mu + m_f)(\gamma_\nu p_2^\nu - m_f)] \\
&= \mathrm{Tr}[\gamma_\mu \gamma_\nu p_1^\mu p_2^\nu - m_f \gamma_\mu p_1^\mu + m_f \gamma_\nu p_2^\nu - m_f^2] \\
&= p_1^\mu p_2^\nu tr[\gamma_\mu \gamma_\nu] - 4m_f^2 \\
&= 4g_{\mu\nu} p_1^\mu p_2^\nu - 4m_f^2 \\
&= 4(p_1 \cdot p_2 - m_f^2).
\end{aligned}$$

and

$$\sum_{s,s'} |\mathcal{M}|^2 = G_F m_f^2 \sqrt{2} \cdot 4(p_1 \cdot p_2 - m_f^2).$$

Since $E_1 = E_2 = M_H/2$, and $\mathbf{p}_1 = -\mathbf{p}_2$, the term $p_1 \cdot p_2 - m_f^2$ can be written as

$$\begin{aligned}
p_1 \cdot p_2 - m_f^2 &= E_1 E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2 - m_f^2 \\
&= E_1^2 + \mathbf{p}_1^2 - m_f^2 \\
&= E_1^2 + (E_1^2 - m_f^2) - m_f^2 \\
&= 2(E_1^2 - m_f^2) \\
&= 2\left(\frac{M_H^2}{4} - m_f^2\right) \\
&= \frac{M_H^2}{2}\left(1 - 4\frac{m_f^2}{M_H^2}\right)
\end{aligned}$$

Therefore, the scattering amplitude is

$$\sum_{s,s'} |\mathcal{M}|^2 = 2G_F M_H^2 m_f^2 \sqrt{2}\left(1 - 4\frac{m_f^2}{M_H^2}\right). \tag{4.9}$$

On the other hand from the kinematics of the problem we have $\mathbf{p}_1^2 = \mathbf{p}_2^2$ y $E_{\mathcal{A}} = E_1 + E_2$. In this way

$$
\begin{aligned}
\mathbf{p}_1^2 = \mathbf{p}_2^2 &= E_2^2 - m_2^2 \\
&= (E_{\mathcal{A}} - E_1)^2 - m_2^2 \\
&= E_{\mathcal{A}}^2 - 2E_{\mathcal{A}}E_1 + E_1^2 - m_2^2 \\
&= E_{\mathcal{A}}^2 - 2E_{\mathcal{A}}E_1 + \mathbf{p}_1^2 + m_1^2 - m_2^2 \\
0 &= E_{\mathcal{A}}^2 - 2E_{\mathcal{A}}E_1 + m_1^2 - m_2^2 \\
E_1 &= \frac{1}{2E_{\mathcal{A}}}\left(E_{\mathcal{A}}^2 + m_1^2 - m_2^2\right) \\
(\mathbf{p}_1^2 + m_1^2)^{1/2} &= \frac{1}{2E_{\mathcal{A}}}\left(E_{\mathcal{A}}^2 + m_1^2 - m_2^2\right) \\
\mathbf{p}_1^2 &= \frac{1}{4E_{\mathcal{A}}^2}\left(E_{\mathcal{A}}^2 + m_1^2 - m_2^2\right)^2 - m_1^2.
\end{aligned}
$$

In the case $m_1 = m_2$ we have

$$
\begin{aligned}
\mathbf{p}_1^2 &= \frac{1}{4}E_{\mathcal{A}}^2 - m_1^2 \\
&= \frac{1}{4}E_{\mathcal{A}}^2\left(1 - \frac{m_1^2}{E_{\mathcal{A}}^2}\right) \\
\mathbf{p}_1 &= \frac{1}{2}E_{\mathcal{A}}\left(1 - 4\frac{m_1^2}{E_{\mathcal{A}}^2}\right)^{1/2}.
\end{aligned}
$$

Setting $E_{\mathcal{A}} = m_A = M_H$, and $m_1 = m_f$, we have

$$
\mathbf{p}_1 = \frac{1}{2}M_H\left(1 - 4\frac{m_f^2}{M_H^2}\right)^{1/2}. \tag{4.10}
$$

Similarly, setting $m_{\mathcal{A}} \to M_H$ in Eq. (4.5), and replacing Eqs. (4.9), and (4.10) in Eq. (4.5), we obtain

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_{\mathrm{CM}}} &= \frac{1}{32\pi^2 M_H^2}\frac{1}{2}M_H\left(1 - 4\frac{m_f^2}{M_H^2}\right)^{1/2} 2G_F M_H^2 m_f^2\sqrt{2}\left(1 - 4\frac{m_f^2}{M_H^2}\right) \\
&= \frac{M_H m_f^2 G_F}{16\pi^2\sqrt{2}}\left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2}.
\end{aligned}
$$

After the integration in $d\Omega_{\mathrm{CM}}$ we have

$$
\Gamma_{H\to f\bar{f}} = \frac{M_H m_f^2 G_F}{4\pi\sqrt{2}}\left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2}, \tag{4.11}
$$

In the limit $m_f \ll M_H$ this expression reduces to

$$\Gamma_{H\to f\bar{f}} = \frac{M_H m_f^2 G_F}{4\pi\sqrt{2}}. \tag{4.12}$$